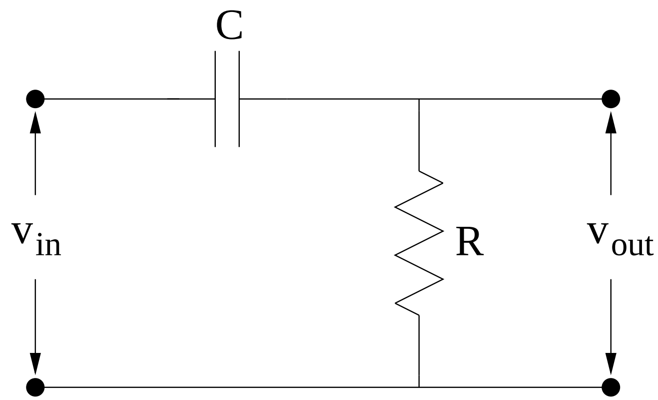
# Filter Circuits

- RC High-Pass Filter
- RC Low-Pass Filter
- Homework

# RC High-Pass Filter




$$i = \frac{v_{in}}{Z} = \frac{v_{in}}{R + \frac{1}{j\omega C}}$$

$$v_{out} = iR = \frac{R}{R + \frac{1}{j\omega C}} v_{in} = \frac{j\omega RC}{1 + j\omega RC} v_{in}$$

$$\frac{v_{out}}{v_{in}} = \frac{j\omega RC}{1 + j\omega RC}$$

$$\left|\frac{v_{out}}{v_{in}}\right| = \sqrt{\left(\frac{v_{out}}{v_{in}}\right)^{*}\left(\frac{v_{out}}{v_{in}}\right)} = \sqrt{\left(\frac{-j\omega RC}{1 - j\omega RC}\right)\left(\frac{j\omega RC}{1 + j\omega RC}\right)} = \sqrt{\frac{(\omega RC)^2}{1 + (\omega RC)^2}} = \frac{\omega RC}{\sqrt{1 + (\omega RC)^2}}$$

# RC High-Pass Filter (cont'd)

It is common practice to express the voltage gain (or attenuation) in decibels (dB) defined as

$$\left|\frac{v_{out}}{v_{in}}\right|_{dB} \equiv 20\log_{10}\left|\frac{v_{out}}{v_{in}}\right|$$

| $\left\|\frac{v_{out}}{v_{in}}\right\|$ | $\left\|\frac{v_{out}}{v_{in}}\right\|_{dB}$ | $\left\|\frac{v_{out}}{v_{in}}\right\|$ | $\left\|\frac{v_{out}}{v_{in}}\right\|_{dB}$ |
|---|---|---|---|
| 0.01 | -40 | 0.5 | -6 |
| 0.1 | -20 | 0.707 | -3 |
| 1.0 | 0 | 1.414 | +3 |
| 10.0 | +20 | 2.0 | +6 |
| 100.0 | +40 | 4.0 | +12 |

# RC High-Pass Filter (cont'd)

- Note that as $\omega \to \infty$, $\left|\frac{v_{out}}{v_{in}}\right| = \frac{\omega RC}{\sqrt{1+(\omega RC)^2}} \to 1.0$, so we can write the high-frequency approximation of the gain as $\left|\frac{v_{out}}{v_{in}}\right|_{high} \cong 1.0$
- Also note that the low frequency approximation of the gain is $\left|\frac{v_{out}}{v_{in}}\right|_{low} \cong \omega RC \propto \omega$
- If we make a log-log graph of $\left|\frac{v_{out}}{v_{in}}\right|$versus $\omega$, we see that the actual gain can be approximated quite well by two straight lines corresponding to $\left|\frac{v_{out}}{v_{in}}\right|_{low}$ and $\left|\frac{v_{out}}{v_{in}}\right|_{high}$ with a sharp break or "knee" given by

$$\left|\frac{v_{out}}{v_{in}}\right|_{low} = \left|\frac{v_{out}}{v_{in}}\right|_{high}$$

$$\omega_B RC = 1$$

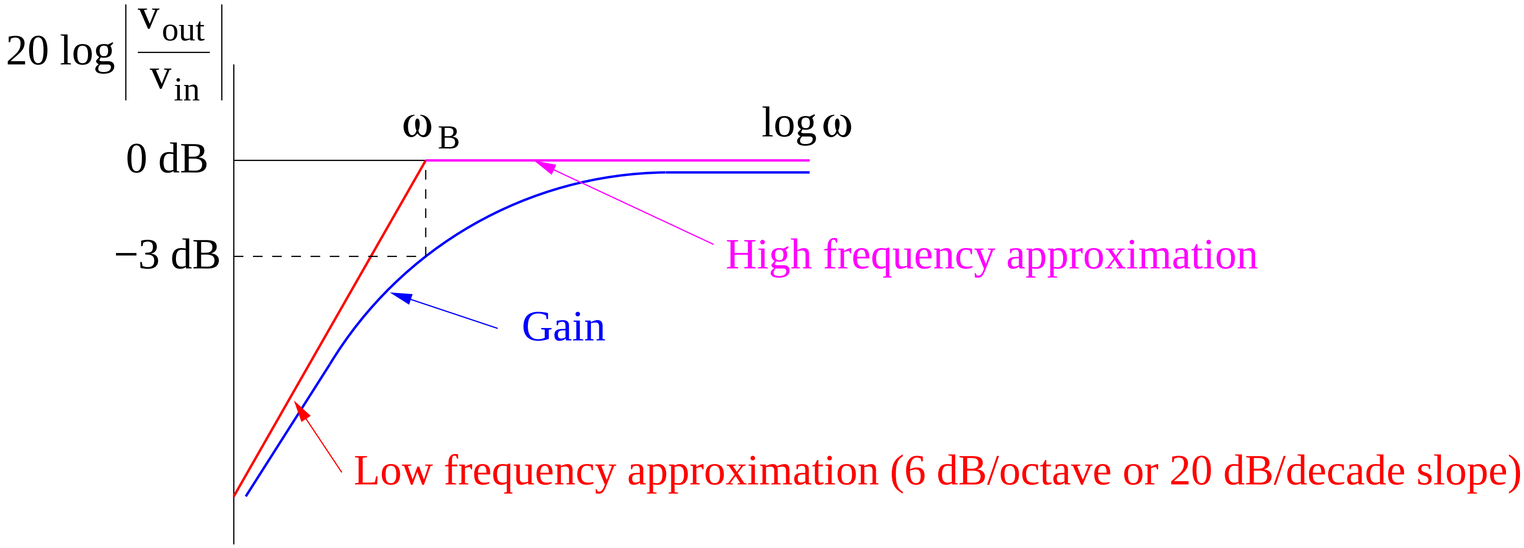
$$\omega_B = \frac{1}{RC}$$

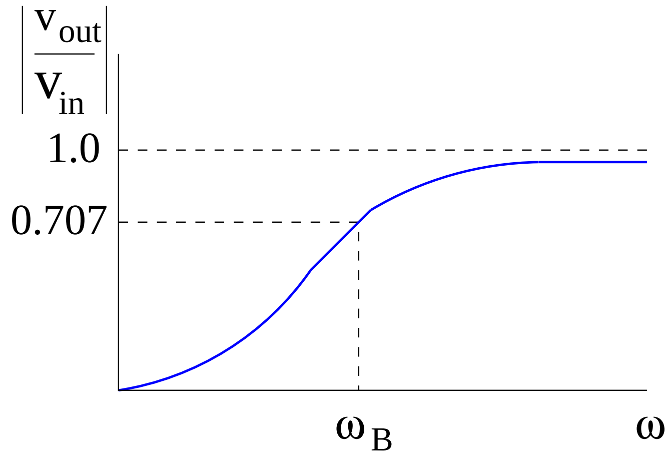
- At this break point, the magnitude of the gain is

$$\left|\frac{v_{out}}{v_{in}}\right| = \frac{\omega_B RC}{\sqrt{1+(\omega_B RC)^2}} = \frac{1}{\sqrt{2}} = 0.707$$

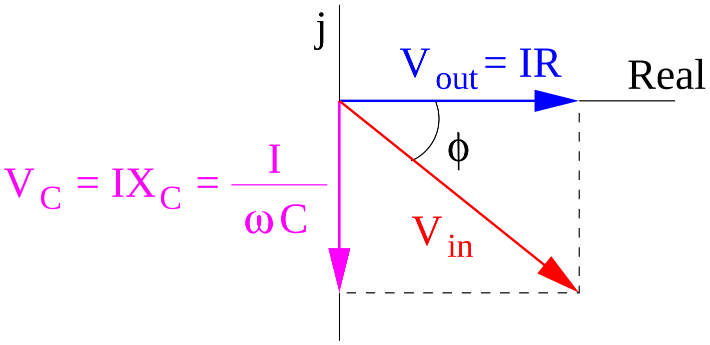
$$\left|\frac{v_{out}}{v_{in}}\right|_{dB} = -3\,dB$$

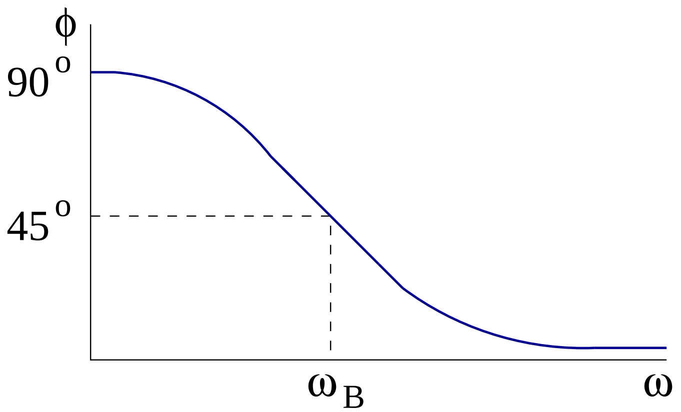
# RC High-Pass Filter (cont'd)



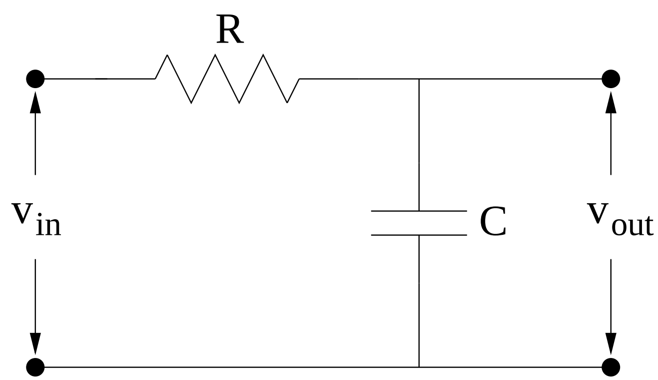



# RC High-Pass Filter (cont'd)



# RC Low-Pass Filter




$$i = \frac{v_{in}}{Z} = \frac{v_{in}}{R + \frac{1}{j\omega C}}$$

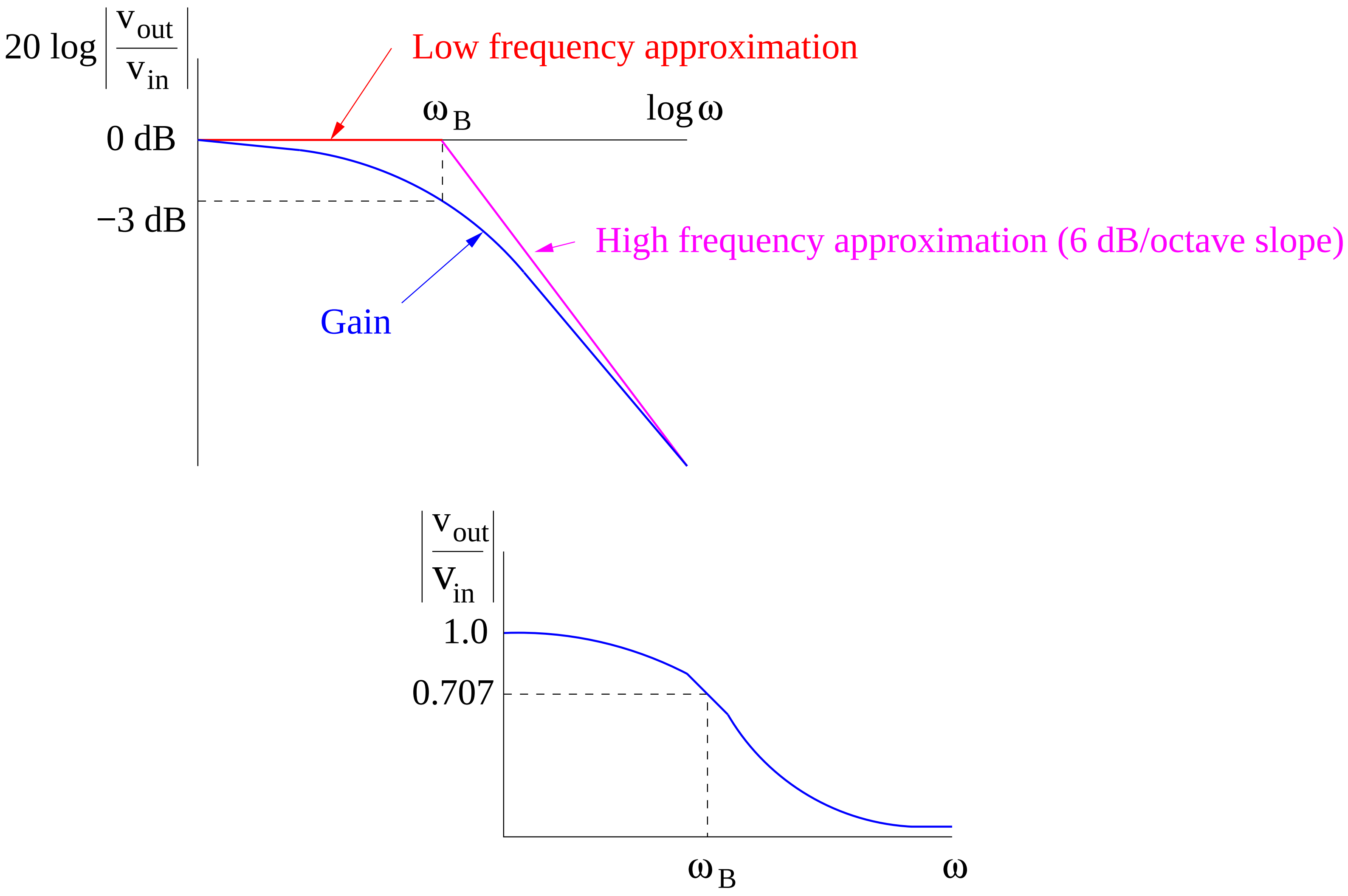
$$v_{out} = iZ_C = \frac{i}{j\omega C} = \frac{\frac{1}{j\omega C}}{R + j\omega C} v_{in} = \frac{1}{1 + j\omega RC} v_{in}$$

$$\frac{v_{out}}{v_{in}} = \frac{1}{1 + j\omega RC}$$

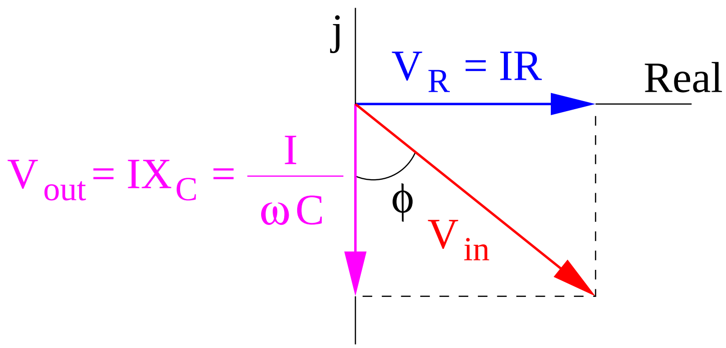
$$\left|\frac{v_{out}}{v_{in}}\right| = \sqrt{\left(\frac{v_{out}}{v_{in}}\right)^* \left(\frac{v_{out}}{v_{in}}\right)} = \sqrt{\left(\frac{1}{1 - j\omega RC}\right)\left(\frac{1}{1 + j\omega RC}\right)} = \frac{1}{\sqrt{1 + (\omega RC)^2}}$$

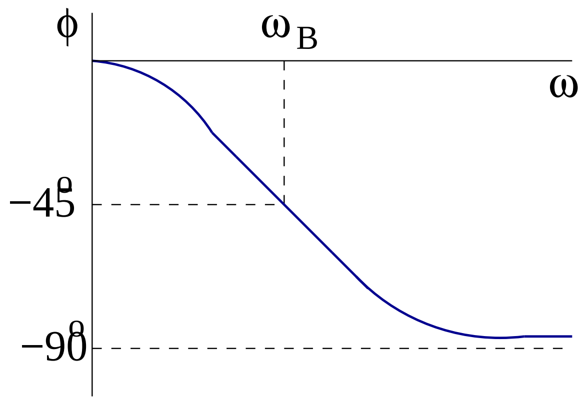
Note that this goes to unity as $\omega \rightarrow 0$, and goes to zero as $\omega \rightarrow \infty$

# RC Low-Pass Filter (cont'd)



# RC Low-Pass Filter (cont'd)



# Homework Set 18 - Due Wed. Feb. 25

1. Design a high-pass RC filter with a breakpoint at 100 kHz. Use a 1-kΩ resistor. Explain in words why the high-pass filter attenuates the low frequencies.
2. Design a low-pass RC filter that will attenuate a 60-Hz sinusoidal voltage by 12 dB relative to the dc gain. Use a 100-Ω resistor. Explain in words why the low-pass filter attenuates the high frequencies.
3. For a low-pass RC filter prove that at the frequency $\omega = 1/RC$ the voltage gain equals 0.707.